18.022 Recitation Handout
19 November 2014

1. (Open Courseware, 18.022 Fall 2010, Homework #12) Let $\mathbf{F} : \mathbb{R}^3 \to \mathbb{R}^3$ be the vector field given by $\mathbf{F}(x, y, z) = ay^2\mathbf{i} + 2y(x + z)\mathbf{j} + (by^2 + z^2)\mathbf{k}$.

(a) For which values of a and b is the vector field $\mathbf{F}$ conservative?

(b) Find a function $f : \mathbb{R}^3 \to \mathbb{R}$ such that $\mathbf{F} = \nabla f$ for these values.

(c) Find an equation describing a surface S with the property that for every smooth oriented curve C lying on S,

$$\int_C \mathbf{F} \cdot d\mathbf{s} = 0,$$

for these values.

2. Find the area of the rectangle $D = [0, a] \times [0, b]$ using Green's theorem.

3. (6.3.19 in *Colley*) Show that the line integral

$$\int_C \frac{x\,dx + y\,dy}{\sqrt{x^2 + y^2}}$$

is path-independent, and evaluate it along the semicircular arc from $(2, 0)$ to $(-2, 0)$.